

Rescattering effects in $\overline{B}_{u,d,s} \rightarrow DP, \overline{D}P$ decays

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(Dated: December 12, 2007)

Abstract

We study quasi-elastic rescattering effects in $\overline{B}_{u,d,s} \rightarrow DP, \overline{D}P$ decays, where P is a light pseudoscalar. The updated measurements of $\overline{B}_{u,d} \rightarrow DP$ decays are used to extract the effective Wilson coefficients $a_1^{\text{eff}} \simeq 0.90$, $a_2^{\text{eff}} \simeq 0.23$, three strong phases $\delta \simeq 53^\circ$, $\theta \simeq 18^\circ$, $\sigma \simeq -88^\circ$, and the mixing angle $\tau \simeq 9^\circ$. This information is used to predict rates of nineteen $\overline{B}_s \rightarrow DP$ and $\overline{B}_{u,d,s} \rightarrow \overline{D}P$ decay modes, including modes of interests in the γ/ϕ_3 program. Many decay rates are found to be enhanced. In particular, the $\overline{B}_s \rightarrow D^0 K^0$ rate is predicted to be 8×10^{-4} , which could be measured soon. The rescattering effects on the corresponding $\overline{B}_{u,d,s} \rightarrow \overline{D}P, DP$ amplitude ratios r_B, r_{B_s} , and the relative strong phases δ_B, δ_{B_s} are studied. Although the decay rates are enhanced in most cases, r_{B,B_s} values are similar to factorization expectation.

PACS numbers: 11.30.Hv, 13.25.Hw, 14.40.Nd

I. INTRODUCTION

Color-suppressed $b \rightarrow c$ decays $\overline{B}^0 \rightarrow D^{(*)0}\pi^0$ [1, 2], $D^0\eta, D^0\omega$ [1], $D^0\eta'$ [3], $D_s^+K^-$ and $D^0\overline{K}^0$ [4, 5] started to emerge in 2001 (for updated measurements, see [6, 7]), with branching ratios that are significantly larger than earlier theoretical expectations based on naive factorization. When combined with color-allowed $\overline{B} \rightarrow D^{(*)}\pi$ modes in an SU(2) framework, the enhancement in the $D^{(*)0}\pi^0$ rate indicates the presence of non-vanishing strong phases, which has attracted much attention [8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18]. We proposed [11] a quasi-elastic final state rescattering (FSI) picture, where the enhancement of color-suppressed D^0h^0 modes is due to rescattering from the color-allowed $D^+\pi^-$ final state. This approach was also applied to study final state interaction in charmless B decays [19].

The quasi-elastic approach was recently extended to $\overline{B} \rightarrow D\overline{K}, \overline{D}\overline{K}$ decays [20]. The color-allowed $B^- \rightarrow D^0K^-$ and color-suppressed $B^- \rightarrow \overline{D}^0K^-$ decays are of interest for the determination of the unitary phase angle ϕ_3 (or $\gamma \equiv \arg V_{ub}^*$, where V is the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix. The Gronau-London-Wyler (GLW) [21], Atwood-Dunietz-Soni (ADS) [22] and “ DK Dalitz plot” [23, 24] methods probe, in varying ways, the interference of the two types of amplitudes in a common final state. The enhancement of color-suppressed DP modes (where P stands for a light pseudoscalar) could imply a larger $\overline{D}K$ rate [15]. Since the strong interaction respects SU(3) and charge conjugation symmetries, the FSI in DP and $\overline{D}P$ modes should be related. It is thus of interest to study DP and $\overline{D}P$ modes together.

Besides making an update with recently available data, we note that data for \overline{B}_s is starting to emerge from the Tevetron [7] and from B factories [25], and we anticipate more to come in the near future, from LHCb and other LHC experiments. Some B_s modes will be useful in the extraction of γ/ϕ_3 [26, 27, 28]. It is thus timely to study \overline{B}_s decays. In this work, we extend the scope of the quasi-elastic rescattering approach to $\overline{B}_s \rightarrow DP, \overline{D}P$ decays, as well as update our previous results using the latest $\overline{B}_{u,d} \rightarrow DP$ data [6, 7].

In Sec. II we briefly summarize and extend the quasi-elastic rescattering formula for $\overline{B}_{u,d,s} \rightarrow DP, \overline{D}P$ decays. Numerical results are reported in Sec. III. The effective Wilson coefficients and rescattering parameters are obtained by using current $\overline{B} \rightarrow DP$ data. By SU(3) symmetry and charge conjugation invariance of the strong interactions, we make predictions on $\overline{B}_s \rightarrow DP$ and $\overline{B}_{u,d,s} \rightarrow \overline{D}P$ rates. The conclusion is then offered in Sec. IV.

An Appendix specifies the source amplitudes used to fit data with rescattering formalism.

II. FINAL STATE RESCATTERING FRAMEWORK

We only briefly summarize, as well as extend, the decay amplitudes obtained in the quasi-elastic approach for $\bar{B} \rightarrow DP, \bar{D}P$ decays, and refer the reader to Refs. [11] and [20] for more detail.

The quasi-elastic strong rescattering amplitudes can be put in four different classes, as given below. For \bar{B} decaying to DP with $C = +1, S = 0, -1$ final states, we have

$$\begin{aligned}
A_{B^- \rightarrow D^0 \pi^- (D^0 K^-)} &= (1 + ir'_0 + ir'_e) A_{B^- \rightarrow D^0 \pi^- (D^0 K^-)}^0, \\
\begin{pmatrix} A_{\bar{B}^0 \rightarrow D^+ K^-} \\ A_{\bar{B}^0 \rightarrow D^0 \bar{K}^0} \end{pmatrix} &= \mathcal{S}_1^{1/2} \begin{pmatrix} A_{\bar{B}^0 \rightarrow D^+ K^-}^0 \\ A_{\bar{B}^0 \rightarrow D^0 \bar{K}^0}^0 \end{pmatrix}, \\
\begin{pmatrix} A_{\bar{B}^0 \rightarrow D^+ \pi^-} \\ A_{\bar{B}^0 \rightarrow D^0 \pi^0} \\ A_{\bar{B}^0 \rightarrow D_s^+ K^-} \\ A_{\bar{B}^0 \rightarrow D^0 \eta_8} \\ A_{\bar{B}^0 \rightarrow D^0 \eta_1} \end{pmatrix} &= \mathcal{S}_2^{1/2} \begin{pmatrix} A_{\bar{B}^0 \rightarrow D^+ \pi^-}^0 \\ A_{\bar{B}^0 \rightarrow D^0 \pi^0}^0 \\ A_{\bar{B}^0 \rightarrow D_s^+ K^-}^0 \\ A_{\bar{B}^0 \rightarrow D^0 \eta_8}^0 \\ A_{\bar{B}^0 \rightarrow D^0 \eta_1}^0 \end{pmatrix}. \tag{1}
\end{aligned}$$

Extending to \bar{B}_s to DP decays with $C = +1, S = 0, +1$ final states, one has

$$\begin{aligned}
\begin{pmatrix} A_{\bar{B}_s^0 \rightarrow D_s^+ \pi^-} \\ A_{\bar{B}_s^0 \rightarrow D^0 K^0} \end{pmatrix} &= \mathcal{S}_1^{1/2} \begin{pmatrix} A_{\bar{B}_s^0 \rightarrow D_s^+ \pi^-}^0 \\ A_{\bar{B}_s^0 \rightarrow D^0 K^0}^0 \end{pmatrix}, \\
\begin{pmatrix} A_{\bar{B}_s^0 \rightarrow D^+ \pi^-} \\ A_{\bar{B}_s^0 \rightarrow D^0 \pi^0} \\ A_{\bar{B}_s^0 \rightarrow D_s^+ K^-} \\ A_{\bar{B}_s^0 \rightarrow D^0 \eta_8} \\ A_{\bar{B}_s^0 \rightarrow D^0 \eta_1} \end{pmatrix} &= \mathcal{S}_2^{1/2} \begin{pmatrix} A_{\bar{B}_s^0 \rightarrow D^+ \pi^-}^0 \\ A_{\bar{B}_s^0 \rightarrow D^0 \pi^0}^0 \\ A_{\bar{B}_s^0 \rightarrow D_s^+ K^-}^0 \\ A_{\bar{B}_s^0 \rightarrow D^0 \eta_8}^0 \\ A_{\bar{B}_s^0 \rightarrow D^0 \eta_1}^0 \end{pmatrix}. \tag{2}
\end{aligned}$$

For $\overline{B}_{u,d} \rightarrow \overline{D}P$ decays with $C = -1$, $S = \pm 1$ final states,

$$\begin{pmatrix} A_{\overline{B}^0 \rightarrow D^- K^+} \\ A_{\overline{B}^0 \rightarrow \overline{D}^0 \overline{K}^0} \end{pmatrix} = \mathcal{S}_1^{1/2} \begin{pmatrix} A_{\overline{B}^0 \rightarrow D^- K^+}^0 \\ A_{\overline{B}^0 \rightarrow \overline{D}^0 \overline{K}^0}^0 \end{pmatrix},$$

$$\begin{pmatrix} A_{B^- \rightarrow \overline{D}^0 K^-} \\ A_{B^- \rightarrow D^- \overline{K}^0} \\ A_{B^- \rightarrow D_s^- \pi^0} \\ A_{B^- \rightarrow D_s^- \eta_8} \\ A_{B^- \rightarrow D_s^- \eta_1} \end{pmatrix} = \mathcal{S}_3^{1/2} \begin{pmatrix} A_{B^- \rightarrow \overline{D}^0 K^-}^0 \\ A_{B^- \rightarrow D^- \overline{K}^0}^0 \\ A_{B^- \rightarrow D_s^- \pi^0}^0 \\ A_{B^- \rightarrow D_s^- \eta_8}^0 \\ A_{B^- \rightarrow D_s^- \eta_1}^0 \end{pmatrix}. \quad (3)$$

And for $\overline{B}_s^0 \rightarrow \overline{D}P$ decays with $C = -1$, $S = 0, +1$ final states,

$$\begin{pmatrix} A_{\overline{B}_s^0 \rightarrow D^- \pi^+} \\ A_{\overline{B}_s^0 \rightarrow \overline{D}^0 \pi^0} \\ A_{\overline{B}_s^0 \rightarrow \overline{D}_s^- K^+} \\ A_{\overline{B}_s^0 \rightarrow \overline{D}^0 \eta_8} \\ A_{\overline{B}_s^0 \rightarrow \overline{D}^0 \eta_1} \end{pmatrix} = \mathcal{S}_2^{1/2} \begin{pmatrix} A_{\overline{B}_s^0 \rightarrow D^- \pi^+}^0 \\ A_{\overline{B}_s^0 \rightarrow \overline{D}^0 \pi^0}^0 \\ A_{\overline{B}_s^0 \rightarrow \overline{D}_s^- K^+}^0 \\ A_{\overline{B}_s^0 \rightarrow \overline{D}^0 \eta_8}^0 \\ A_{\overline{B}_s^0 \rightarrow \overline{D}^0 \eta_1}^0 \end{pmatrix}. \quad (4)$$

In these expressions, the square root of the rescattering S -matrix are denoted as $\mathcal{S}_i^{1/2} = (1 + i\mathcal{T}_i)^{1/2} = 1 + i\mathcal{T}_i'$, with

$$\mathcal{T}_1 = \begin{pmatrix} r_0 & r_e \\ r_e & r_0 \end{pmatrix},$$

$$\mathcal{T}_2 = \begin{pmatrix} r_0 + r_a & \frac{r_a - r_e}{\sqrt{2}} & r_a & \frac{r_a + r_e}{\sqrt{6}} & \frac{\bar{r}_a + \bar{r}_e}{\sqrt{3}} \\ \frac{r_a - r_e}{\sqrt{2}} & r_0 + \frac{r_a + r_e}{2} & \frac{r_a}{\sqrt{2}} & \frac{r_a + r_e}{2\sqrt{3}} & \frac{\bar{r}_a + \bar{r}_e}{\sqrt{6}} \\ r_a & \frac{r_a}{\sqrt{2}} & r_0 + r_a & \frac{r_a - 2r_e}{\sqrt{6}} & \frac{\bar{r}_a + \bar{r}_e}{\sqrt{3}} \\ \frac{r_a + r_e}{\sqrt{6}} & \frac{r_a + r_e}{2\sqrt{3}} & \frac{r_a - 2r_e}{\sqrt{6}} & r_0 + \frac{r_a + r_e}{6} & \frac{\bar{r}_a + \bar{r}_e}{3\sqrt{2}} \\ \frac{\bar{r}_a + \bar{r}_e}{\sqrt{3}} & \frac{\bar{r}_a + \bar{r}_e}{\sqrt{6}} & \frac{\bar{r}_a + \bar{r}_e}{\sqrt{3}} & \frac{\bar{r}_a + \bar{r}_e}{3\sqrt{2}} & \tilde{r}_0 + \frac{\bar{r}_a + \bar{r}_e}{3} \end{pmatrix},$$

$$\mathcal{T}_3 = \begin{pmatrix} r_0 + r_a & r_a & \frac{r_e}{\sqrt{2}} & \frac{r_e - 2r_a}{\sqrt{6}} & \frac{\bar{r}_e + \bar{r}_a}{\sqrt{3}} \\ r_a & r_0 + r_a & -\frac{r_e}{\sqrt{2}} & \frac{r_e - 2r_a}{\sqrt{6}} & \frac{\bar{r}_a + \bar{r}_e}{\sqrt{3}} \\ \frac{r_e}{\sqrt{2}} & -\frac{r_e}{\sqrt{2}} & r_0 & 0 & 0 \\ \frac{r_e - 2r_a}{\sqrt{6}} & \frac{r_e - 2r_a}{\sqrt{6}} & 0 & r_0 + \frac{2}{3}(r_a + r_e) & -\frac{\sqrt{2}}{3}(\bar{r}_a + \bar{r}_e) \\ \frac{\bar{r}_e + \bar{r}_a}{\sqrt{3}} & \frac{\bar{r}_a + \bar{r}_e}{\sqrt{3}} & 0 & -\frac{\sqrt{2}}{3}(\bar{r}_a + \bar{r}_e) & \tilde{r}_0 + \frac{\bar{r}_a + \bar{r}_e}{3} \end{pmatrix}, \quad (5)$$

where r_e , r_a and r_0 are charge exchange, annihilation, singlet exchange rescattering parameters [11], while \bar{r}_i and \tilde{r}_i are those for $D\Pi(8) \leftrightarrow D^0\eta_1$ and $D^0\eta_1 \leftrightarrow D^0\eta_1$ scattering, respectively [20]. SU(3) symmetry requires that \mathcal{T}'_i has the same structure as \mathcal{T}_i . Hence the \mathcal{T}'_i is basically \mathcal{T}_i , but with r_j , \bar{r}_j and \tilde{r}_j replaced by r'_j , \bar{r}'_j and \tilde{r}'_j , respectively. We note that some of the above formulas were already reported in [11, 20], while all formulas for the second and fourth cases, and some for the third case, are new. We have used charge conjugation invariance and SU(3) symmetry of the strong interactions, hence the $r_i^{(\prime)}$, $\bar{r}_i^{(\prime)}$ and $\tilde{r}_i^{(\prime)}$ coefficients in $\mathcal{T}_i^{(\prime)}$ of $\bar{B}_{u,d,s} \rightarrow \bar{D}P$ rescattering amplitudes are identical to those in $\mathcal{T}_i^{(\prime)}$ of $\bar{B}_{u,d,s} \rightarrow DP$ rescattering amplitudes.

Using SU(3) symmetry and $\mathcal{S}^\dagger\mathcal{S} = 1$, the rescattering parameters are given by [20]

$$\begin{aligned} (1 + ir_0) &= \frac{1}{2}(1 + e^{2i\delta}), \\ ir_e &= \frac{1}{2}(1 - e^{2i\delta}), \\ ir_a &= \frac{1}{8}(3\mathcal{U}_{11} - 2e^{2i\delta} - 1), \\ i(\bar{r}_a + \bar{r}_e) &= \frac{3}{2\sqrt{2}}\mathcal{U}_{12}, \\ i(\tilde{r}_0 + \frac{\tilde{r}_a + \tilde{r}_e}{3}) &= \mathcal{U}_{22} - 1, \end{aligned} \tag{6}$$

where

$$\mathcal{U} = \mathcal{U}^T = \begin{pmatrix} \cos \tau & \sin \tau \\ -\sin \tau & \cos \tau \end{pmatrix} \begin{pmatrix} e^{2i\theta} & 0 \\ 0 & e^{2i\sigma} \end{pmatrix} \begin{pmatrix} \cos \tau & -\sin \tau \\ \sin \tau & \cos \tau \end{pmatrix}, \tag{7}$$

and we have set the overall phase factor $(1 + ir_0 + ir_e)$ in \mathcal{S} to unity. This phase convention is equivalent to choosing the $A_{\bar{B}^0 \rightarrow D^0\pi^-}$ amplitude to be real. The r'_i , \bar{r}'_i and \tilde{r}'_i in $\mathcal{S}^{1/2}$ can be obtained by using the above formulas with phases (δ, θ, σ) reduced by half. We need three phases and one mixing angle to specify FSI effects in DP and $\bar{D}P$ rescattering. The interpretation of these phases and mixing angle in term of SU(3) decomposition can be found in [20].

To use the FSI formulas, we need to specify A^0 . We use naive factorization amplitudes A^f for A^0 to avoid double counting of FSI effects [11, 20]. And the explicit forms of A^0 are given in Appendix A. We stress that, in our quasi-elastic rescattering approach, SU(3) symmetry is applied only in the $D\Pi \rightarrow D\Pi$ rescattering matrix, which should hold for m_B scale. Certain amount of SU(3) breaking effects which have to do with meson formation are included in the factorization amplitudes via decay constants and form factors.

TABLE I: Branching ratios of various $\overline{B} \rightarrow DP$ and $D\overline{K}$ modes in 10^{-4} units. The second column is the experimental data [6, 7], which is taken as input. The naive factorization model results are given in third column. Fitting the experimental data shown in the first column with quasi-elastic FSI (fit parameters as given in Table II), we obtain the FSI fit results given in the last column. The factorization results are recovered by setting FSI phases in Table II to zero.

Mode	$\mathcal{B}^{\text{exp}} (10^{-4})$	$\mathcal{B}^{\text{fac}} (10^{-4})$	$\mathcal{B}^{\text{FSI}} (10^{-4})$
$B^- \rightarrow D^0 \pi^-$	48.4 ± 1.5	$48.4^{+5.2}_{-4.2}$	$48.4^{+0.8}_{-0.8}$
$\overline{B}^0 \rightarrow D^+ \pi^-$	26.8 ± 1.3	$31.9^{+1.7}_{-1.8}$	$26.9^{+1.0}_{-1.0}$
$\overline{B}^0 \rightarrow D^0 \pi^0$	2.61 ± 0.24	$0.57^{+0.25}_{-0.14}$	$2.42^{+0.19}_{-0.16}$
$\overline{B}^0 \rightarrow D_s^+ K^-$	0.28 ± 0.05	0	0.26 ± 0.03
$\overline{B}^0 \rightarrow D^0 \eta$	2.02 ± 0.35	$0.33^{+0.14}_{-0.08}$	$2.06^{+0.30}_{-0.29}$
$\overline{B}^0 \rightarrow D^0 \eta'$	1.25 ± 0.23	$0.20^{+0.09}_{-0.05}$	$1.27^{+0.21}_{-0.19}$
$B^- \rightarrow D^0 K^-$	4.02 ± 0.21	$4.01^{+0.49}_{-0.38}$	$4.01^{+0.07}_{-0.09}$
$\overline{B}^0 \rightarrow D^+ K^-$	2.04 ± 0.57	2.43 ± 0.13	1.97 ± 0.07
$\overline{B}^0 \rightarrow D^0 \overline{K}^0$	0.52 ± 0.07	$0.14^{+0.06}_{-0.04}$	$0.60^{+0.03}_{-0.04}$

III. RESULTS

In our numerical study, masses and lifetimes are taken from the Particle Data Group (PDG) [7], and B to charm meson decay branching ratios are taken from [6, 7]. We fix $V_{ud} = 0.97419$, $V_{us} = 0.22568$, $V_{cb} = 0.04166$, $V_{cs} = 0.997334$, $|V_{ub}| = 3.624 \times 10^{-3}$ [29], and use the decay constants $f_\pi = 131$ MeV, $f_K = 156$ MeV [7] and $f_{D(s)} = 200$ (230) MeV.

We have six parameters to describe the processes with rescattering from factorization amplitudes: the two effective Wilson coefficients a_1^{eff} and a_2^{eff} , the three rescattering phases δ , θ and σ , and one mixing angle τ in $\mathcal{S}^{1/2}$. These parameters are fitted with rates of nine \overline{B} decay to $C = 1, S = 0, -1$ modes, namely $\overline{B} \rightarrow D^+ \pi^-, D^0 \pi^-, D^0 \pi^0, D^0 \eta, D^0 \eta', D_s^+ K^-, D^0 K^-, D^+ K^-$ and $D^0 \overline{K}^0$ decays, given in Table I. The fitted FSI parameters are listed in Table II. We then use the extracted parameters to predict nineteen $\overline{B}_s \rightarrow DP$ (Table III) and $\overline{B}_{u,d,s} \rightarrow \overline{D}P$ (Table IV) decays. Predictions on the ratios of $\overline{B} \rightarrow \overline{D}P$ and $\overline{B} \rightarrow DP$ amplitudes are also given.

The errors of the fitted parameters given in Table II are propagated from the experimental

TABLE II: Fit parameters in the SU(3) FSI picture, where results are from using $\overline{B} \rightarrow D^0\pi^-$, $D^+\pi^-$, $D^0\pi^0$, $D^0\eta$, $D^0\eta'$, $D_s^+K^-$, D^0K^- , D^+K^- and $D^0\overline{K}^0$ decay rates (Table I) as fit input. There is a two fold ambiguity (the overall sign of phases) in the solutions. The SU(3) phases and mixing angle are reexpressed in terms of the rescattering parameters r'_i , \bar{r}'_i , \tilde{r}'_i .

parameter	result	parameter	result
χ^2_{\min}	1.92	$\chi^2_{\min}/\text{d.o.f.}$	0.64
a_1^{eff}	0.90 ± 0.02	a_2^{eff}	$0.23^{+0.03}_{-0.02}$
δ	$\pm(52.9^{+1.9}_{-2.0})^\circ$	θ	$\pm(17.8^{+3.0}_{-2.9})^\circ$
σ	$\mp(87.7^{+27.7}_{-27.3})^\circ$	τ	$(9.2^{+5.2}_{-2.7})^\circ$
$1 + ir'_0$	$(0.80 \pm 0.01) \pm (0.40 \pm 0.01)i$	ir'_e	$(0.20 \pm 0.01) \mp (0.40 \pm 0.01)i$
ir'_a	$(0.07 \pm 0.01) \mp (0.10 \pm 0.01)i$	$i(\bar{r}'_a + \bar{r}'_e)$	$(-0.15^{+0.04}_{-0.03}) \mp (0.22^{+0.08}_{-0.09})i$
$1 + i\tilde{r}'_0 + i\frac{\tilde{r}'_e + \tilde{r}'_a}{3}$	$(0.06 \pm 0.46) \mp (0.97^{+0.17}_{-0.01})i$		

errors by requiring $\chi^2 \leq \chi^2_{\min} + 1$. The fitted values of these parameters are similar to those in our previous analysis [20]¹. There is a two fold ambiguity (the overall sign of the phases) in the solutions. We obtain $\chi^2_{\min}/\text{d.o.f.} = 0.64$ indicating a good fit to these modes. The effective Wilson coefficients $a_{1,2}^{\text{eff}}$ are close to expectation [30, 31]. From $|r'_e| > |r'_a|$ we infer that exchange rescattering is dominant over annihilation rescattering.

We show in the fourth column of Table I the fit output for the nine fitted $\overline{B} \rightarrow DP$ and $D\overline{K}$ modes. These fitted branching ratios (in units of 10^{-4}) should be compared with data and naive factorization results given in the second and third columns. The FSI results reproduce the data quite well, as it should. The errors for the FSI results are from data only. The factorization results can be recovered by using the parameters of Table II but with FSI phases set to zero. Note that unitarity is implied automatically, i.e. sum of rates within coupled modes are unchanged by FSI.

Our main interest here is the color-suppressed B_s decays. The predicted branching ratios of various $\overline{B}_s \rightarrow DP$ modes with $C = +1$, $S = 0, +1$ final states are shown in Table III, where the second column gives naive factorization results and the third column gives the FSI

¹ We found and corrected a numerical error in our previous analysis, resulting in the value of σ taking opposite sign.

TABLE III: The predictions on branching ratios of various $\overline{B}_s \rightarrow DP$ modes in 10^{-4} and 10^{-5} units, respectively. The errors for the FSI results are from $\overline{B} \rightarrow DP$ data only.

Mode	\mathcal{B}^{exp}	$\mathcal{B}^{\text{fac}} (10^{-4})$	$\mathcal{B}^{\text{FSI}} (10^{-4})$
$\overline{B}_s^0 \rightarrow D_s^+ \pi^-$	30 ± 7	$30.5^{+1.6}_{-1.7}$	$24.9^{+0.8}_{-0.9}$
$\overline{B}_s^0 \rightarrow D^0 K^0$	–	$2.2^{+1.0}_{-0.6}$	7.9 ± 0.5
Mode	$\mathcal{B}^{\text{exp}} (10^{-5})$	$\mathcal{B}^{\text{fac}} (10^{-5})$	$\mathcal{B}^{\text{FSI}} (10^{-5})$
$\overline{B}_s^0 \rightarrow D^+ \pi^-$	–	0	$0.16^{+0.03}_{-0.02}$
$\overline{B}_s^0 \rightarrow D^0 \pi^0$	–	0	0.08 ± 0.01
$\overline{B}_s^0 \rightarrow D_s^+ K^-$	–	$23.2^{+1.2}_{-1.3}$	$19.5^{+0.7}_{-0.7}$
$\overline{B}_s^0 \rightarrow D^0 \eta$	–	$0.6^{+0.3}_{-0.2}$	$2.9^{+0.5}_{-0.5}$
$\overline{B}_s^0 \rightarrow D^0 \eta'$	–	$0.9^{+0.4}_{-0.2}$	2.0 ± 0.6

results. Again, the factorization results are recovered by using the same parameters of Table II but with FSI phases set to zero, and the errors for the FSI results are from $\overline{B} \rightarrow DP$ data only. Analogous to $\overline{B}^0 \rightarrow D^0 \pi^0$ enhancement being fed from $\overline{B}^0 \rightarrow D^+ \pi^-$ rescattering, it is interesting to note that $\overline{B}_s^0 \rightarrow D_s^+ \pi^-$ with FSI rescattering to $D^0 K^0$, brings $\overline{B}_s^0 \rightarrow D^0 K^0$ rate to the 10^{-3} level, which can be measured soon. This is helped by the absence of annihilation rescattering. The $\overline{B}_s \rightarrow D^0 \eta$, $D^0 \eta'$ modes are the direct analogs of $\overline{B}^0 \rightarrow D^0 \pi^0$. One can see that their rates are brought up to levels similar to $\overline{B}^0 \rightarrow D^0 \pi^0$. Rescattering slightly reduces the $\overline{B}_s \rightarrow D_s^+ K^-$ and $B_s \rightarrow D_s^+ \pi^-$ rates. The $D_s^+ K$ mode will be used to extract γ/ϕ_3 at LHCb [27], while the $D^0 \eta$, $D^0 \eta'$ modes could also be useful [28].

$\overline{B}_{u,d,s} \rightarrow \overline{D}P$ decays are V_{ub} suppressed, and mostly not measured yet, except $D_s^- \pi^+$. The quasi-elastic FSI formalism allows us to make predictions even for such modes that are color-suppressed. Predictions for $\overline{B}_{u,d,s} \rightarrow \overline{D}P$ decays are shown in Table IV, where again, experimental results and limits [7, 33] are shown in the second column, and the third and fourth columns are naive factorization and FSI results, respectively. Same comments on FSI parameters apply. Agreement of the only observed $\overline{B}^0 \rightarrow D_s^- \pi^+$ mode with theoretical prediction is improved by including FSI effects. $\overline{B}_s \rightarrow D_s^- K^+$ is slightly reduced, but overall, the redistribution of decay rates by rescattering is not very significant.

TABLE IV: Predictions for $\overline{B}_{u,d,s} \rightarrow \overline{D}P$ rates. Experimental results and limits [7, 33] are shown in the second column, and naive factorization and FSI results are given in the third and fourth columns.

Mode	$\mathcal{B}^{\text{exp}} (10^{-5})$	$\mathcal{B}^{\text{fac}} (10^{-5})$	$\mathcal{B}^{\text{FSI}} (10^{-5})$
$B^- \rightarrow \overline{D}^0 K^-$	–	0.2 ± 0.1	$0.3^{+0.1}_{-0.3}$
$B^- \rightarrow D^- \overline{K}^0$	< 0.5	0	0.03 ± 0.01
$B^- \rightarrow D_s^- \pi^0$	< 20	0.9 ± 0.1	0.8 ± 0.0
$B^- \rightarrow D_s^- \eta$	< 50	0.5 ± 0.0	0.4 ± 0.1
$B^- \rightarrow D_s^- \eta'$	–	0.3 ± 0.0	0.5 ± 0.1
$\overline{B}^0 \rightarrow D_s^- \pi^+$	1.4 ± 0.3	1.7 ± 0.1	$1.4^{+0.0}_{-0.1}$
$\overline{B}^0 \rightarrow \overline{D}^0 K^0$	–	0.2 ± 0.1	0.5 ± 0.0
$\overline{B}_s^0 \rightarrow D^- \pi^+$	–	0	0.02 ± 0.00
$\overline{B}_s^0 \rightarrow \overline{D}^0 \pi^0$	–	0	0.01 ± 0.00
$\overline{B}_s^0 \rightarrow D_s^- K^+$	–	2.3 ± 0.1	2.0 ± 0.1
$\overline{B}_s^0 \rightarrow \overline{D}^0 \eta$	–	$0.06^{+0.03}_{-0.02}$	$0.3^{+0.0}_{-0.1}$
$\overline{B}_s^0 \rightarrow \overline{D}^0 \eta'$	–	$0.09^{+0.04}_{-0.02}$	0.2 ± 0.1

Combining $\overline{B}_s \rightarrow D_s^+ K^-$ and $\overline{B}_s \rightarrow D_s^- K^+$, we can compare our predicted ratio

$$R \equiv \frac{\mathcal{B}(\overline{B}_s \rightarrow D_s^\pm K^\mp)}{\mathcal{B}(\overline{B}_s \rightarrow D_s^+ \pi^-)} = 0.090 \pm 0.002, \quad (8)$$

with the recent experimental result of $R = 0.107 \pm 0.019 \pm 0.008$ [32] from CDF. The agreement is reasonable. In fact, the larger rescattering of $\overline{B}_s \rightarrow D_s^+ \pi^-$ to $\overline{B}_s \rightarrow D^0 K^0$ has helped enhance the ratio from the lower factorization value.

In Table V, we compare our predictions for various $\overline{B}_s \rightarrow DP$ rates with results obtained in other approaches [34, 35] that differ in the application of SU(3) symmetry. Most of our results agree with others. For modes with $\eta^{(\prime)}$, our results are closer to those in [34] obtained using earlier data. In both approaches, U(3) symmetry is not imposed and $D\eta_1$ is treated as an independent component. Although predictions on $\overline{B}_s \rightarrow DP$ rates are similar in all three works, it should be note that there is a major difference between ours and the other two's approaches. In this work, the information obtained in $\overline{B}_{u,d} \rightarrow DP$ rescattering from data is used to predict not only $\overline{B}_s \rightarrow DP$ decays [via SU(3) symmetry], but also $\overline{B}_{u,d,s} \rightarrow \overline{D}P$

TABLE V: Comparison of predictions for branching ratios of various $\overline{B}_s \rightarrow DP$ modes to other approaches.

$\mathcal{B} (10^{-4})$	This work	CF [34]	CS [35]
$\overline{B}_s^0 \rightarrow D_s^+ \pi^-$	$24.9^{+0.8}_{-0.9}$	29 ± 6	22 ± 1
$\overline{B}_s^0 \rightarrow D^0 K^0$	7.9 ± 0.5	8.1 ± 1.8	5.3 ± 0.3
$\mathcal{B} (10^{-5})$	This work	CF [34]	CS [35]
$\overline{B}_s^0 \rightarrow D^+ \pi^-$	$0.16^{+0.03}_{-0.02}$	0.20 ± 0.06	0.14 ± 0.03
$\overline{B}_s^0 \rightarrow D^0 \pi^0$	0.08 ± 0.01	0.10 ± 0.03	0.07 ± 0.01
$\overline{B}_s^0 \rightarrow D_s^+ K^-$	19.5 ± 0.7	18 ± 3	20 ± 1
$\overline{B}_s^0 \rightarrow D^0 \eta$	2.9 ± 0.5	2.1 ± 1.2	1.4 ± 0.1
$\overline{B}_s^0 \rightarrow D^0 \eta'$	2.1 ± 0.6	0.98 ± 0.76	2.9 ± 0.2

decays [through charge conjugation invariance of the S -matrix]. SU(3) symmetry itself is not sufficient to relate $\overline{B}_{u,d,s} \rightarrow DP$ and $\overline{B}_{u,d,s} \rightarrow \overline{D}P$ amplitudes. Hence, in the two other works, which employed solely SU(3) symmetry to decay amplitudes, no prediction on $\overline{B}_{u,d,s} \rightarrow \overline{D}P$ decays were given by analyzing $\overline{B}_{u,d} \rightarrow DP$ data.

In Table VI, $r_{B(B_S)}$, $\delta_{B(B_S)}$ for various modes are predicted and compared with data, where the amplitude ratio $r_{B(s)}$ and the strong phase difference $\delta_{B(s)}$ are defined as

$$r_{B(s)}(DP) = \left| \frac{A(\overline{B}_{(s)} \rightarrow \overline{D}\overline{P})}{A(\overline{B}_{(s)} \rightarrow DP)} \right|, \quad \delta_{B(s)}(DP) = \arg \left[\frac{e^{i\phi_3} A(\overline{B}_{(s)} \rightarrow \overline{D}\overline{P})}{A(\overline{B}_{(s)} \rightarrow DP)} \right]. \quad (9)$$

The weak phase ϕ_3 is removed from $A(\overline{B}_{(s)} \rightarrow \overline{D}\overline{P})$ in defining $\delta_{B(s)}$. Except $r_B(D^0 K^0)$ the effects from final state interaction are mild. We see that our $r_B(D^0 K^-)$ and $\delta_B(D^0 K^-)$ agree with the Dalitz analysis results of Belle and BaBar. Our $r_B(D^0 K^-)$ is also in agreement with the fit from UT_{fit} group obtained by using all three methods of GLW, ADS and DK Dalitz analysis [37].

IV. CONCLUSION

We study quasi-elastic rescattering effects in $\overline{B}_{u,d,s} \rightarrow DP, \overline{D}P$ modes. The updated data for nine $\overline{B}_{u,d} \rightarrow DP$ modes are used to extract $a_{1,2}^{\text{eff}}$ and four rescattering parameters. We find the effective Wilson coefficients $a_1^{\text{eff}} \simeq 0.90$, $a_2^{\text{eff}} \simeq 0.23$, the strong phases $\delta \simeq 54^\circ$,

TABLE VI: Naive factorization and FSI results on $r_{B(s)}$, $\delta_{B(s)}$ with $|V_{ub}| = 3.67 \times 10^{-3}$, and compared to the experimental results [36, 37]. The errors for the FSI results are from DP data only.

	Expt	fac	FSI
$r_B(D^0 K^-)$	$0.16 \pm 0.05 \pm 0.01 \pm 0.05$ (Belle) < 0.14 (1σ) (BaBar) 0.071 ± 0.024 (UT _{fit})	0.07 ± 0.01	0.09 ± 0.01
$\delta_B(D^0 K^-)$	$(146^{+19}_{-20} \pm 3 \pm 23)^\circ$ (Belle) $(118 \pm 63 \pm 19 \pm 36)^\circ$ (BaBar)	180°	$180^\circ \mp (39.4^{+5.4}_{-6.4})^\circ$
$r_B(D^0 K^0)$	–	0.38 ± 0.00	0.29 ± 0.01
$\delta_B(D^0 K^0)$	–	180°	$180^\circ \pm (8.6^{+0.7}_{-0.5})^\circ$
$r_{B_s}(D_s^+ K^-)$	–	0.38 ± 0.00	0.38 ± 0.00
$\delta_{B_s}(D_s^+ K^-)$	–	180°	$180^\circ \pm (0.1 \pm 0.0)^\circ$
$r_{B_s}(D^0 \eta)$	–	0.38 ± 0.00	0.38 ± 0.00
$\delta_{B_s}(D^0 \eta)$	–	180°	$180^\circ \mp (0.6 \pm 0.0)^\circ$
$r_{B_s}(D^0 \eta')$	–	0.38 ± 0.00	0.38 ± 0.00
$\delta_{B_s}(D^0 \eta')$	–	180°	$180^\circ \mp (0.2 \pm 0.1)^\circ$

$\theta \simeq 18^\circ$, $\sigma \simeq -88^\circ$ and mixing angle $\tau \simeq 9^\circ$. Since strong interaction respects SU(3) symmetry and charge conjugation symmetry, the formalism can be used to predict $\overline{B}_s \rightarrow DP$ and $\overline{B}_{u,d,s} \rightarrow \overline{D}P$ rates and $r_{B(B_s)}$, without refers to any experimental information on $\overline{B}_{u,d,s} \rightarrow \overline{D}P$ decays, which is limited by the smallness of the corresponding decay rates.

Our results are summarized as following: (a) The $\overline{B}_s^0 \rightarrow D^0 K^0, D^0 \eta, D^0 \eta'$ rates are enhanced in the presence of FSI. In particular, the $\overline{B}_s^0 \rightarrow D^0 K^0$ rate is close to 10^{-3} level and can be measured soon. (b) The predicted $\overline{B}_s \rightarrow D_s^+ \pi^-$ rate and the ratio of $\mathcal{B}(\overline{B}_s \rightarrow D_s^\pm K^\mp)/\mathcal{B}(\overline{B}_s \rightarrow D_s^+ \pi^-)$ is in better agreement with experimental results. (c) Except the $\overline{B}^0 \rightarrow D^0 K^0$ mode, the FSI effects on $r_{B(B_s)}$ are mild. (d) The predicted $r_B(D^0 K^-)$ agree with data and the fit from the UTfit collaboration, while $\delta_B(D^0 K^-)$ agree with data.

Acknowledgments

We would like to thank Alexey Drutskoy for useful discussion. This work is supported in part by the National Science Council of R.O.C. under Grants NSC-95-2112-M-033-MY2 and NSC96-2112-M-002-022.

APPENDIX A: EXPLICIT EXPRESSION OF A^0

As mentioned in the text, we use naive factorization amplitudes A^f for A^0 to avoid double counting of FSI effects. For each final state, we have

$$\begin{aligned}
A_{B^- \rightarrow D^0 \pi^-}^f &= V_{cb} V_{ud}^* (T_f + C_f), & A_{\bar{B}^0 \rightarrow D^+ \pi^-}^f &= V_{cb} V_{ud}^* (T_f + E_f), \\
A_{\bar{B}^0 \rightarrow D^0 \pi^0}^f &= \frac{V_{cb} V_{ud}^*}{\sqrt{2}} (-C_f + E_f), & A_{\bar{B}^0 \rightarrow D_s^+ K^-}^f &= V_{cb} V_{ud}^* E_f, \\
A_{\bar{B}^0 \rightarrow D^0 \eta_8}^f &= \frac{V_{cb} V_{ud}^*}{\sqrt{6}} (C_f + E_f), & A_{\bar{B}^0 \rightarrow D^0 \eta_1}^f &= \frac{V_{cb} V_{ud}^*}{\sqrt{3}} (C_f + E_f), \\
A_{B^- \rightarrow D^0 K^-} &= V_{cb} V_{us}^* (T_f + C_f), & A_{\bar{B}^0 \rightarrow D^+ K^-} &= V_{cb} V_{us}^* T_f, \\
A_{\bar{B}^0 \rightarrow D^0 \bar{K}^0} &= V_{cb} V_{us}^* C_f, & A_{\bar{B}_s^0 \rightarrow D_s^+ \pi^-}^f &= V_{cb} V_{ud}^* T_f, \\
A_{\bar{B}_s^0 \rightarrow D^0 K^0}^f &= V_{cb} V_{ud}^* C_f, & A_{\bar{B}_s^0 \rightarrow D^+ \pi^-}^f &= V_{cb} V_{us}^* E_f, \\
A_{\bar{B}_s^0 \rightarrow D^0 \pi^0}^f &= \frac{V_{cb} V_{us}^*}{\sqrt{2}} E_f, & A_{\bar{B}_s^0 \rightarrow D^0 \eta_8}^f &= \frac{V_{cb} V_{us}^*}{\sqrt{6}} (-2C_f + E_f), \\
A_{\bar{B}_s^0 \rightarrow D^0 \eta_1}^f &= \frac{V_{cb} V_{us}^*}{\sqrt{3}} (C_f + E_f), & A_{\bar{B}_s^0 \rightarrow D_s^+ K^-}^f &= V_{cb} V_{us}^* (T_f + E_f),
\end{aligned} \tag{A1}$$

where the super- and subscripts f indicate naive factorization amplitude, and

$$\begin{aligned}
T_f &= \frac{G_F}{\sqrt{2}} a_1^{\text{eff}} (m_B^2 - m_D^2) f_P F_0^{BD}(m_P^2), \\
C_f &= \frac{G_F}{\sqrt{2}} a_2^{\text{eff}} (m_B^2 - m_P^2) f_D F_0^{BP}(m_D^2), \\
E_f &= \frac{G_F}{\sqrt{2}} a_2^{\text{eff}} (m_D^2 - m_P^2) f_B F_0^{0 \rightarrow DP}(m_B^2).
\end{aligned} \tag{A2}$$

$F_0^{BD(BP)}$ is the $\bar{B}_{u,d,s} \rightarrow D_{u,d,s}(P)$ transition form factor and $F_0^{0 \rightarrow DP}$ is the vacuum to DP (time-like) form factor.

TABLE VII: Form factors taken from [39, 40]. For $B_{(s)} \rightarrow \eta'$ form factors the mixing angle and Clebsch-Gordan coefficients are included [see Eq. (A6)].

Form factor	value	Form factor	value
$F_0^{B\pi}(m_{D,D_s}^2)$	0.28	$F_0^{B_{(s)}D_{(s)}}(m_{\pi,K}^2)$	0.67
$F_0^{B\eta}(m_{D,D_s}^2)$	0.15	$F_0^{B\eta'}(m_{D,D_s}^2)$	0.13
$F_0^{BK}(m_{D,D_s}^2)$	0.43	$F_0^{BsK}(m_D^2)$	0.40
$F_0^{Bs\eta}(m_D^2)$	-0.29	$F_0^{Bs\eta'}(m_{D,D_s}^2)$	0.35

For $\overline{B}_{u,d,s} \rightarrow \overline{D}_{u,d,s}P$ decays, we have,

$$\begin{aligned}
A_{B^- \rightarrow \overline{D}^0 K^-}^f &= V_{ub}V_{cs}^* (c_f + a_f), & A_{B^- \rightarrow D^- \overline{K}^0}^f &= V_{ub}V_{cd}^* a_f, \\
A_{B^- \rightarrow \overline{D}^0 \eta_8}^f &= \frac{V_{ub}V_{cs}^*}{\sqrt{6}}(t_f - 2a_f), & A_{B^- \rightarrow \overline{D}^0 \eta_1}^f &= \frac{V_{ub}V_{cd}^*}{\sqrt{3}}(t_f + a_f), \\
A_{B^- \rightarrow D_s^- \pi^0}^f &= \frac{V_{ub}V_{cs}^*}{\sqrt{2}}t_f, & A_{\overline{B}^0 \rightarrow D_s^- \pi^+} &= V_{ub}V_{cs}^* t_f, \\
A_{\overline{B}^0 \rightarrow \overline{D}^0 \overline{K}^0} &= V_{ub}V_{cs}^* c_f, & A_{\overline{B}^0 \rightarrow D^- \pi^+}^f &= V_{ub}V_{cs}^* e_f, \\
A_{\overline{B}^0 \rightarrow \overline{D}^0 \pi^0}^f &= \frac{V_{ub}V_{cs}^*}{\sqrt{2}} e_f, & A_{\overline{B}^0 \rightarrow \overline{D}^0 \eta_8}^f &= \frac{V_{ub}V_{cs}^*}{\sqrt{6}} (-2c_f + e_f), \\
A_{\overline{B}^0 \rightarrow \overline{D}^0 \eta_1}^f &= \frac{V_{ub}V_{cs}^*}{\sqrt{3}} (c_f + e_f), & A_{\overline{B}^0 \rightarrow D_s^- K^+}^f &= V_{ub}V_{cs}^* (t_f + e_f),
\end{aligned} \tag{A3}$$

where, as before, the super- and subscripts f indicate naive factorization amplitude, and

$$\begin{aligned}
t_f &= \frac{G_F}{\sqrt{2}} a_1^{\text{eff}} (m_B^2 - m_P^2) f_D F_0^{BP}(m_D^2), \\
c_f &= \frac{G_F}{\sqrt{2}} a_2^{\text{eff}} (m_B^2 - m_P^2) f_D F_0^{BP}(m_D^2), \\
e_f &= \frac{G_F}{\sqrt{2}} a_2^{\text{eff}} (m_P^2 - m_D^2) f_B F_0^{PD}(m_B^2).
\end{aligned} \tag{A4}$$

Note that we have $(t_f, e_f) = (T_f, E_f)$ with D and P interchanged and $c_f = C_f$ (without the interchange of D and P).

The $D^0 \eta_8$ and $D^0 \eta_1$ are not physical final states. The physical η, η' mesons are defined through

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos \vartheta & -\sin \vartheta \\ \sin \vartheta & \cos \vartheta \end{pmatrix} \begin{pmatrix} \eta_8 \\ \eta_1 \end{pmatrix}, \tag{A5}$$

with the mixing angle $\vartheta = -15.4^\circ$ [38]. Form factors are taken from [39, 40], where we list the relevant values in Table VII. For $B_{(s)} \rightarrow \eta'$ form factors the mixing angle and

Clebsch-Gordan coefficients are included,

$$\begin{aligned}
F^{B\eta}(m_{D,D_s}^2) &= \left(\frac{\cos \vartheta}{\sqrt{6}} - \frac{\sin \vartheta}{\sqrt{3}} \right) F_0^{B\pi}(m_{D,D_s}^2), \\
F^{B\eta'}(m_{D,D_s}^2) &= \left(\frac{\sin \vartheta}{\sqrt{6}} + \frac{\cos \vartheta}{\sqrt{3}} \right) F_0^{B\pi}(m_{D,D_s}^2) \\
F^{B_s\eta}(m_{D,D_s}^2) &= \left(-2 \frac{\cos \vartheta}{\sqrt{6}} - \frac{\sin \vartheta}{\sqrt{3}} \right) F_0^{B_s\eta_s}(m_{D,D_s}^2), \\
F^{B_s\eta'}(m_{D,D_s}^2) &= \left(-2 \frac{\sin \vartheta}{\sqrt{6}} + \frac{\cos \vartheta}{\sqrt{3}} \right) F_0^{B_s\eta_s}(m_{D,D_s}^2),
\end{aligned} \tag{A6}$$

where η_s is the $s\bar{s}$ component of η and η' and the form factor $F^{B\pi}(m_{D,D_s}^2)$ and $F_0^{B_s\eta_s}(m_{D,D_s}^2)$ are taken from [39] and [40], respectively.

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